

Math 5B Final Exam – K. Hogue
Spring 2021 v1
200 POINTS

Instructions on Canvas

FILL IN THE BLANK WITH THE MOST APPROPRIATE ANSWER. NO PARTIAL CREDIT. (4 POINTS EACH)

(1) TRUE OR FALSE: If $0 \leq a_n \leq b_n$ and $\sum a_n$ diverges, then $\sum b_n$ diverges _____

(2) Express the point $(-\sqrt{3}, 1)$ in polar coordinates(exactly)_____

(3) $\int \frac{1}{1+x^2} dx =$ _____

(4) $\frac{d}{dx} \sin^{-1}(3x) =$ _____

(5) $\frac{d}{dx} \left(\frac{x^3}{\ln(5x)} \right) =$ _____ (simplify)

(6) $\int e^{4x} dx =$ _____

(7) Given $\sum_{n=1}^{\infty} a_n$, if $\lim_{n \rightarrow \infty} a_n \neq 0$ what is known about the convergence/divergence of the series? _____

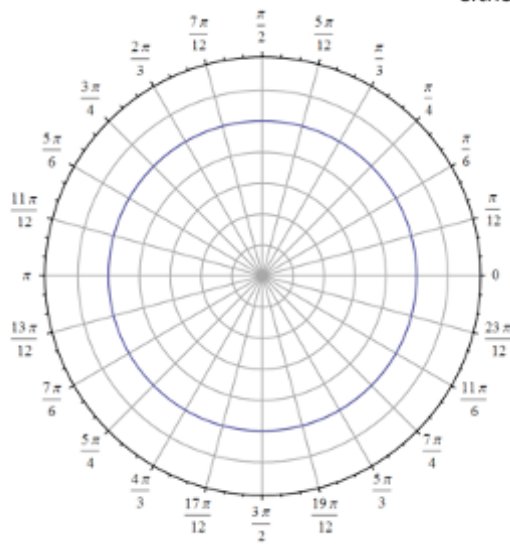
(8) If $f(x) = 5 - \ln x$, find $f^{-1}(x)$ _____

(9) $\lim_{x \rightarrow 0^+} x^2 \ln x =$ _____ (show work clearly)

(10) Find the sum exactly: $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n}$ _____

- (11) Solve the differential equation $y' = x^4 y$ with initial condition $y(0) = 3$. Solve for y explicitly. Be sure to show constants carefully. (11 points)

- (12) (a). Graph the polar curve $r = 2 - 2\sin\theta$ Label two *polar* points ON the graph
 (b). Find the area of the portion of the graph in the first quadrant. (22 points)



(13)

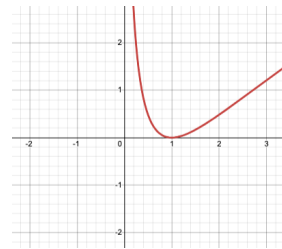
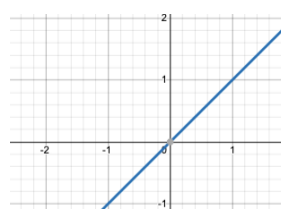
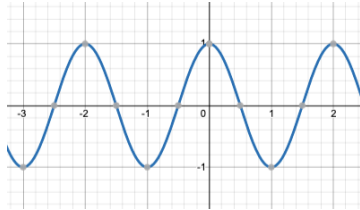
(12 points)

Match the graphs of the parametric pair $x(t)$ and $y(t)$ on the left with the graph in the xy plane on the right.

$x(t)$

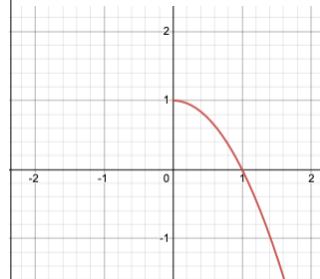
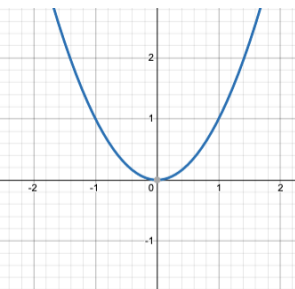
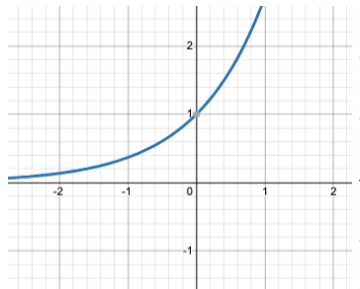
$y(t)$

(a) _____



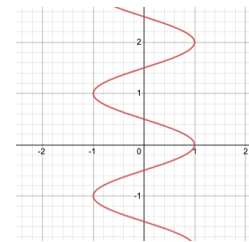
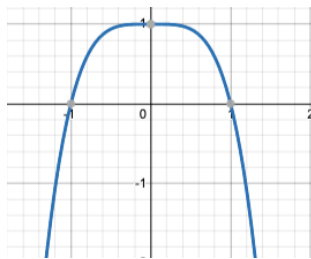
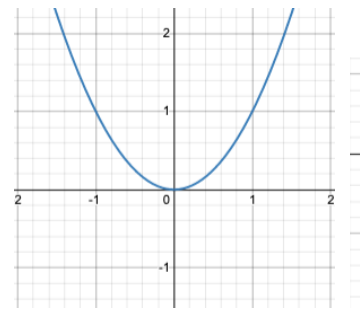
A

(b) _____



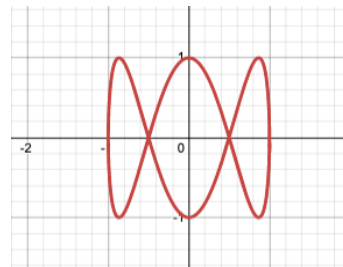
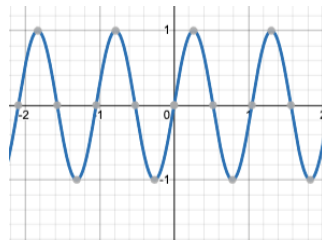
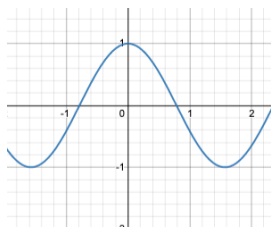
B

(c) _____



C

(d) _____



D

(14) For each of the following series, classify as convergent (absolute or conditional if applicable) or divergent. SHOW ALL DETAILS. (15 points each)

(a)
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

(b)
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{5^n}{(3n)!}$$

(15) Clearly show the integral test applies to the following series and use it to determine whether the series converges or diverges. Correct mathematical notation is expected. (20 points)

$$\sum_{n=1}^{\infty} n^2 e^{-n}$$

(16) Compute each of the following integrals:

(a) $\int_0^1 \frac{x^3}{\sqrt{4-x^2}} dx$ You must use trigonometric substitution on this one. No credit for a different method
(20 points)

(b) $\int_e^5 \frac{1}{x(\ln x)^2} dx$ (10 points)

- (17) (a). Find the equation of a tangent line to $f(x) = e^x$ which contains the origin (10 points)
(b) Sketch $f(x)$ and the tangent line from part (a).

(18) (a) Use series to approximate $\int_0^{1/2} x \cos(x^2) dx$ with error less than 0.00001 .(15 points)

(b) Find the value of the integral exactly by integrating directly. (10 points)